

**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

Cambridge Ordinary Level

## **MARK SCHEME for the May/June 2015 series**

### **4037 ADDITIONAL MATHEMATICS**

**4037/21**

Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2015 series for most Cambridge IGCSE<sup>®</sup>, Cambridge International A and AS Level components and some Cambridge O Level components.

® IGCSE is the registered trademark of Cambridge International Examinations.

Page 2	Mark Scheme	Syllabus	Paper
	Cambridge O Level – May/June 2015	4037	21

### Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1	(a)	$\frac{\log_3 x}{\log_3 27}$ $\frac{\log_3 x}{3} \text{ isw}$	M1	Can use other interim bases if all correct but M1 when in base 3 only
	(b)	$\log_a 15 - \log_a 3 = \log_a 5 \text{ soi}$ $\log_a 5^3 \text{ or } \log_a a$ $\log_a y = \log_a 125a \Rightarrow y = 125a$	M1 M1 A1	NOT $\log_3 x \div 3$
2	(a)	$[f(x) = ]2x - 4 \text{ and } [f(x) = ]-2x + 4$	B1,B1	Condone $y = \dots$
	(b)		B1 B1 B1	correct shape; $y$ intercept marked or seen nearby; intent to tend to $y = 3$ (i.e. not tending to or cutting $x$ -axis)
3	(a)	$\mathbf{A} = \frac{1}{4} \left[ \begin{pmatrix} 51 & -8 & 19 \\ 31 & 2 & 65 \end{pmatrix} - \begin{pmatrix} 20 & 0 & -5 \\ 15 & -10 & 25 \end{pmatrix} \right]$ $\mathbf{A} = \begin{pmatrix} 8 & -2 & 6 \\ 4 & 3 & 10 \end{pmatrix}$	M1 A1	Integer values
	(b) (i)	The (total) value of the stock in <b>each</b> of the 3 shops	B1	Must have “each” oe
	(ii)	The <b>total</b> value of the stock in all 3 shops	B1	Must have “total” oe

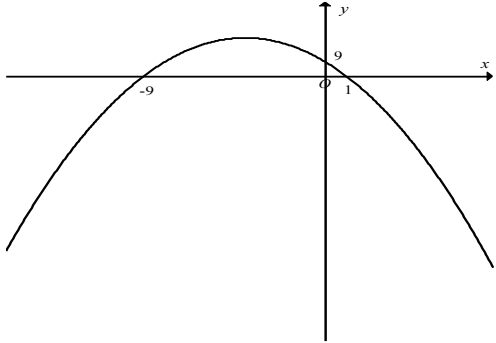
Page 3	Mark Scheme	Syllabus	Paper
	Cambridge O Level – May/June 2015	4037	21

4	(i)	$\frac{PT}{8} = \tan\left(\frac{3\pi}{8}\right)$ oe  $PT = 19.3$	M1  A1	$\frac{PT}{\sin\frac{3\pi}{8}} = \frac{8}{\sin\frac{\pi}{8}}$  awrt 19.3
	(ii)	$\frac{1}{2} \times 8^2 \times \frac{3\pi}{4}$ oe (75.4)  $8 \tan\left(\frac{3\pi}{8}\right) \times 8 - \text{their sector}$ oe (=154.5-'75.4')  79.1	M1  M1  A1	or $\frac{1}{2} \times 8^2 \times \frac{3\pi}{8}$  $8 \times \text{their } PT - \text{their sector}$  awrt 79.1
	(iii)	$8\left(\frac{3\pi}{4}\right)$ oe (18.8)  $\left[6\pi + 16 \tan\left(\frac{3\pi}{8}\right)\right] = 57.5$	M1  A1	Accept 57.4 to 57.5
5	(a)	Permutation because the order matters oe	B1	
	(b) (i)	${}^6C_4 + {}^5C_4 + {}^7C_4$ 55	M1 A1	3 correct terms added
	(ii)	${}^2C_1 \times {}^6C_1 \times {}^5C_1 \times {}^7C_1$ 420	M1 A1	4 correct terms multiplied
(iii)	${}^6C_3 \times {}^2C_1$ or ${}^2C_2 \times {}^5C_1 \times {}^6C_1$ summation 70	M1 M1 A1	for either correct product adding two correct products  If 0 scored, then SC1 for 1,1,1,0 and 0,0,2,1 seen	
6	(i)	$2t^2 - 14t + 12 = 0$ $(t-1)(t-6)$ oe $(t=) 1$	M1  A1	Can use formula, etc.  If $t = 1$ with no working, then M1A1
	(ii)	$\int (2t^2 - 14t + 12) dt$ $(s=) \frac{2t^3}{3} - \frac{14t^2}{2} + 12t$	M1  A2,1,0	-1 for each error or for +c left in or limits introduced
	(iii)	$(a=) \frac{dv}{dt} (4t - 14)$ $[4(3) - 14 =] -2$ cao	M1  A1	

Page 4	Mark Scheme	Syllabus	Paper
	Cambridge O Level – May/June 2015	4037	21

7	(a)	$\vec{AB} = 15\mathbf{b} - 5\mathbf{a} = 5(3\mathbf{b} - \mathbf{a})$ or $\vec{BC} = 24\mathbf{b} - 3\mathbf{a} - 15\mathbf{b} = 3(3\mathbf{b} - \mathbf{a})$ or $\vec{AC} = 24\mathbf{b} - 3\mathbf{a} - 5\mathbf{a} = 8(3\mathbf{b} - \mathbf{a})$	B1 B1	Any correct simplified vector Any second simplified vector
		Comment: e.g. the vectors are scalar multiples of each other AND they have a common point ( $A$ , $B$ or $C$ as appropriate)	B1dep	Dep on both B marks being awarded.
	(b) (i)	$2\mathbf{i} + 11\mathbf{j}$ soi $\Rightarrow \sqrt{2^2 + 11^2}$ $\sqrt{125}$ or $5\sqrt{5}$ or 11.2 (3 s.f.) or better	B1 B1fT	ft <i>their</i> $2\mathbf{i} + 11\mathbf{j}$ (not $\vec{OP}$ or $\vec{OQ}$ )
	(ii)	$\frac{1}{5\sqrt{5}} (2\mathbf{i} + 11\mathbf{j})$ isw	B1fT	ft <i>their</i> answers from (i)
	(iii)	$\frac{\mathbf{i} - 4\mathbf{j} + 3\mathbf{i} + 7\mathbf{j}}{2}$ or $\mathbf{i} - 4\mathbf{j} + \frac{2\mathbf{i} + 11\mathbf{j}}{2}$ or $3\mathbf{i} + 7\mathbf{j} - \frac{2\mathbf{i} + 11\mathbf{j}}{2}$ $2\mathbf{i} + 1.5\mathbf{j}$	M1 A1	
8	(a) (i)	$k e^{4x+3}$ (+c) oe $k = \frac{1}{4}$ oe	M1 A1	any constant, non-zero $k$
	(ii)	$\frac{1}{4} (e^{4(3)+3} - e^{4(2.5)+3})$ or better 706 650.99... = 707 000 to 3 sf or better	DM1 A1	ft <i>their</i> integral attempt Accept $\frac{1}{4}(e^{15} - e^{13})$
	(b) (i)	$k \sin\left(\frac{x}{3}\right)$ (+c) $k = 3$	M1 A1	any constant, non-zero $k$
	(ii)	$3 \sin\left(\frac{\pi}{6} \times \frac{1}{3}\right) - 3 \sin(0)$ 0.520 944... = 0.521 to 3 sf or better	DM1 A1	Dep on <i>their</i> integral attempt in sin; condone omission of lower limit Accept $3 \sin\left(\frac{\pi}{18}\right)$
	(c)	$\int (x^{-2} + 2 + x^2) dx = \frac{x^{-1}}{-1} + 2x + \frac{x^3}{3}$ $+ c$	B1 M1 A1 B1	Expands – accept unsimplified integration of <i>their</i> 3 term expansion Fully correct $+c$

Page 5	Mark Scheme	Syllabus	Paper
	Cambridge O Level – May/June 2015	4037	21

<p><b>9 (a)</b></p> <p><math>(4x-1)(x+5) [\leq 0]</math></p> <p>critical values <math>\frac{1}{4}</math> and <math>-5</math> soi</p> <p><math>-5 \leq x \leq \frac{1}{4}</math></p> <p><b>(b) (i)</b></p> <p><math>(x+4)^2 - 25</math> or <math>a = 4</math> and <math>b = -25</math></p> <p><b>(ii)</b></p> <p>(Greatest value <math>\Rightarrow</math>) 25</p> <p><math>x = -4</math></p> <p><b>(iii)</b></p> 	<p>M1</p> <p>A1</p> <p>A1</p> <p>B1, B1</p> <p>B1ft</p> <p>B1ft</p> <p>B1</p> <p>B1</p>	<p>Solves quadratic</p> <p>Accept: <math>\left[-5, \frac{1}{4}\right]; -5 \leq x</math> AND <math>x \leq 0.25</math></p> <p>Must be clear</p> <p>Correct shape with maximum in second quadrant and crossing positive and negative axes correctly</p> <p>All 3 intercepts correctly shown on graph</p>
<p><b>10 (i)</b></p> <p><math>\ln y = \ln(Ab^x) \Rightarrow \ln y = \ln A + \ln b^x</math></p> <p><math>\Rightarrow \ln y = \ln A + x \ln b</math></p> <p><b>(ii)</b></p> <p><math>\ln A = 11.4 \Rightarrow A = e^{\text{their } 11.4}</math></p> <p><math>A = 90\,000</math> cao</p> <p><math>\ln b = -1</math></p> <p><math>b = 0.4</math> cao</p> <p><b>(iii)</b></p> <p><math>x = 2.5 \Rightarrow \ln y = 9</math></p> <p><math>y = e^9</math> or 8000 to 1 sf</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>condone misread of scale for M1 (11.2 only)</p> <p>Allow awrt -1</p> <p>Allow awrt 8100</p>
<p><b>11 (i)</b></p> <p><math>7 - x, x, 6 - x</math> oe</p> <p><i>their</i> attempt at <math>7 - x + x + 6 - x + 16 = 25</math> oe</p> <p><math>x = 4</math></p> <p><b>(ii)</b></p> <p><math>23 - y, y, 9 - y</math> oe</p> <p><math>48 = 30 + 25 + 15 - 7 - 6 - (\text{their } 4 + y) + \text{their } 4</math> oe soi</p> <p><math>y = 9</math></p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>Condone <math>x = 4</math> for all 3 marks</p> <p>or <math>n(A \cup C) = 48 - 16 = 32</math></p> <p>or <math>32 = 30 + 15 - (\text{their } 4 + y)</math></p> <p>or <math>48 = (23 - y) + 3 + 16 + y + 4 + 2 + (9 - y)</math></p> <p>Condone <math>y = 9</math> for all 3 marks</p>
<p><b>(iii)</b></p> <p><math>n(C) = 15</math> and <math>y + n(B \cap C) = 9 + 6 = 15</math></p> <p>[and so <math>A' \cap B' \cap C = \emptyset</math>].</p>	<p>B1</p>	<p>or equivalent deduction</p>